## **Semiconductors and Semiconductor Devices**

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# **Formation of Energy Bands**

The origin of the band structure for the typical elemental semiconductors is due to a mixing of the s- and p-wavefunctions, tetrahedral bonding orbitals  $(sp^3)$  are formed. The bonding orbitals constitute the valence band and the antibonding orbitals the conduction band. The size of the band gap must be temperature dependent. With increasing temperature the lattice parameter increases due to thermal expansion. The splitting between the bonding and antibonding states therefore decreases and the band gap becomes smaller.



Energy gaps of	<sup>-</sup> Si	and	Ge
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	Е <sub>g</sub> ( <i>T</i> = 0 К) (eV)	E <sub>g</sub> (T = 300 K) (eV)		
Si	1.17	1.1		
Ge	0.75	0.67		

Interatomic distance

## Metal, Semiconductor and Insulator

Completely filled bands and completely empty bands do not contribute to electrical conductivity, and only a partly filled electronic band can contribute to electric current.



If the distance between the upper edge of the highest filled band (valence band) and the lower edge of the lowest empty band (conduction band) is not too large (e.g.  $\sim 1 \text{ eV}$ ), then a small fraction of the states in the vicinity of the upper edge of the valence band is unoccupied and the corresponding electrons are found in the conduction band.

## **Energy Bands of Germanium and Silicon**



## **Compound Semiconductors**

Based on the *sp*<sup>3</sup> bonding, one can correctly identify another important class, the III-V semiconductors, which are compound semiconductors comprising elements from the third and fifth groups of the periodic table. Typical examples are InSb, InAs, InP, GaP, GaAs, GaSb, and AlSb. In these compound crystals the bonding is mixed ionic and covalent.



In contrast to the elemental semiconductors, the most important representatives of the III-V semiconductors possess a so-called direct band gap.

	<i>E</i> <sub>g</sub> (0 K) [eV]	<i>E</i> <sub>g</sub> (300 K) [eV]	$m_n^*/m$	$m_{ m lh}^*/m$	$m^*_{ m hh}/m$	$m_{ m soh}^{st}/ m m$	⊿ [eV]
GaAs	1.52	1.43	0.07	0.08	0.5	0.15	0.34
GaSb	0.81	0.7	0.047	0.05	0.3	0.14	0.8
InSb	0.24	0.18	0.015	0.02	0.4	0.11	0.8
[nAs	0.43	0.35	0.026	0.025	0.4	0.14	0.4
[nP	1.42	1.35	0.073	0.12	0.6	0.12	0.11

#### **Intrinsic Carrier Concentrations**

For a semiconductor, we want to know the concentration of intrinsic carries as function of temperature T, in terms of its band gap  $E_{g}$ .

The energy of an electron in the conduction band is  $\epsilon_k = E_c + \hbar^2 k^2 / 2m_e$ The density of electron states at  $\epsilon$  is  $D_e(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2}\right)^{3/2} (\epsilon - E_c)^{1/2}$ Fermi-Dirac distribution for  $\epsilon - \mu \gg k_B T$  reduces to  $f_e \simeq \exp[(\mu - \epsilon)/k_B T]$ The concentration of electrons in the conduction band is



The distribution function  $f_h$  for holes is related to the electron distribution by

$$\begin{split} f_h &= 1 - \frac{1}{\exp[(\boldsymbol{\epsilon} - \boldsymbol{\mu})/k_B T] + 1} = \frac{1}{\exp[(\boldsymbol{\mu} - \boldsymbol{\epsilon})/k_B T] + 1} \\ &\cong \exp[(\boldsymbol{\epsilon} - \boldsymbol{\mu})/k_B T] \ , \quad \text{ if } (\boldsymbol{\mu} - \boldsymbol{\epsilon}) \gg k_B T. \end{split}$$

The density of hole states at  $\epsilon$  is

$$D_h(m{\epsilon}) = rac{1}{2 \pi^2} \!\! \left( rac{2 m_h}{\hbar^2} 
ight)^{\!\!\!3/2} \! (E_v - m{\epsilon})^{1/2} \; ,$$

The concentration p of holes in the valence band is

$$p = \int_{-\infty}^{E_c} D_h(\epsilon) f_h(\epsilon) d\epsilon = 2 \left( \frac{m_h k_B T}{2\pi \hbar^2} \right)^{3/2} \exp[(E_v - \mu)/k_B T]$$

We multiply together the expressions for *n* and *p* to obtain the equilibrium relation

$$np = 4 \left( \frac{k_B T}{2\pi \hbar^2} \right)^3 (m_c m_h)^{3/2} \exp(-E_g/k_B T) , \quad E_g = E_c - E_v$$

The product of *np* is constant at a given temperature.

For intrinsic carriers,  $n_i$  and  $p_i$ ,

$$n_i = p_i = 2 \left( \frac{k_B T}{2\pi \hbar^2} \right)^{3/2} (m_e m_h)^{3/4} \exp(-E_g/2k_B T)$$

To obtain the Fermi level  $\mu$ , we start from  $n_i = p_i$ , so

$$n_{i} = 2\left(\frac{m_{e}k_{B}T}{2\pi\hbar^{2}}\right)^{3/2} \exp[(\mu - E_{c})/k_{B}T] = 2\left(\frac{m_{h}k_{B}T}{2\pi\hbar^{2}}\right)^{3/2} \exp[(E_{v}-\mu)/k_{B}T] = \rho_{i}$$

then,

$$\begin{split} \exp(2\mu/k_B T) &= (m_h/m_e)^{3/2} \quad \exp[(E_v + E_c)/kT] \\ \mu &= 1/2 (E_v + E_c) - \frac{3}{4} k_B T \ln (m_h/m_e) \ . \end{split}$$

If  $m_h = m_e$ , then  $\mu = 1/2 (E_v + E_c)$  and the Fermi level is in the middle of the forbidden gap.

## **Impurity Conductivity**

Certain impurities and imperfections drastically affect the electrical properties of a semiconductor. The addition of boron to silicon in the proportion of 1 boron atom to 10<sup>5</sup> silicon atoms increases the conductivity of pure at room temperature by a factor of 10<sup>3</sup>. The deliberate addition of impurities to a semiconductor is called **doping**.

We consider the effect of impurities in silicon and germanium. These elements crystallize in the diamond structure. Each atom forms four covalent bonds, one with each of its nearest neighbors, corresponding to the chemical valence four. If an impurity atom of valence five, such as phosphorus, arsenic, or antimony, is substituted in the lattice in place of a normal atom, there will be one valence electron from the impurity atom left over after the four covalent bonds are established with the nearest neighbors, that is, after the impurity atom has been accommodated in the structure with as little disturbance as possible. Impurity atoms that can give up an electron are called **donors**.

#### **Donor States**

The impurity atoms of valence five such as P, As, and Sb are called **donors** because they donate electrons to the conduction band in order to complete the covalent bonds with neighbor atoms, leaving electrons in the band.



The extra electron moves in the coulomb potential  $e/\epsilon r$  of the impurity ion, where  $\epsilon$  in a covalent crystal is the static dielectric constant of the medium.

We estimate the ionization energy of the donor impurity. The Bohr theory of the hydrogen atom may be modified to take into account the dielectric constant of the medium and the effective mass of an electron in the periodic potential of the crystal. The ionization energy of atomic hydrogen is  $-e^4m/(2\hbar^2)$ .

In the semiconductor with dielectric constant  $\epsilon$  we replace  $e^2$  by  $e^2/\epsilon$  and m by the effective mass  $m_e$  to obtain

$$E_d = \frac{e^4 m_e}{2\epsilon^2 \hbar^2} = \left(\frac{13.6}{\epsilon^2} \frac{m_e}{m}\right) eV \; ; \qquad \text{and} \qquad a_d = \frac{\epsilon \hbar^2}{m_e e^2} = \left(\frac{0.53\epsilon}{m_e/m}\right) \text{\AA}$$

 $E_d$  is the ionization energy and  $a_d$  the Bohr radius of the donor.

To obtain a general impression of the impurity levels we use  $m_e \simeq 0.1 m$ for electrons in germanium and  $m_e \simeq 0.2 m$  in silicon. The static dielectric constant  $\epsilon$  is 15.8 for Ge and 11.7 for Si. Then, we obtain

$$E_d = 5 \text{ meV}$$
 and  $a_d = 80 \text{ Å}$  for Ge;  
 $E_d = 20 \text{ meV}$  and  $a_d = 30 \text{ Å}$  for Si.

#### **Acceptor States**



Trivalent impurities such as B, Al, Ga, and In are called **acceptors** because they accept electrons from the valence band in order to complete the covalent bonds with neighbor atoms, leaving holes in the band.

## **Impurity Carrier Concentrations**

For a doped semiconductor, we want to know the concentration of conductive carries as function of temperature T and impurity energy levels  $E_d$  and  $E_a$ .

Neutrality condition demands  $n + N_A^- = p + N_D^+$ , in which

$$N_{\rm D} = N_{\rm D}^0 + N_{\rm D}^+$$
, and  $n_{\rm D} = N_{\rm D}^0 = N_{\rm D} [1 + \exp(E_{\rm D} - E_{\rm F})/\& T]^{-1}$ ,  
 $N_{\rm A} = N_{\rm A}^0 + N_{\rm A}^-$ .  $p_{\rm A} = N_{\rm A}^0 = N_{\rm A} [1 + \exp(E_{\rm F} - E_{\rm A})/\& T]^{-1}$ .



## **Properties of Holes**

Holes are vacant orbits in a band whose properties are important in an almost filled band.

A hole acts in applied electric and magnetic fields as if it has a positive charge +*e*. The reason is given in five steps that follow:

1. Crystal momentum:  $\mathbf{k}_{h} = -\mathbf{k}_{e}$ 

The total wavevector of the electrons in a filled band is zero:  $\Sigma \mathbf{k} = 0$ , where the sum is over all states in a Brillouin zone. This result follows from the geometrical symmetry of the Brillouin zone: every fundamental lattice type has symmetry under the inversion operation  $\mathbf{r} - \mathbf{r}$  about any lattice point; it follows that the Brillouin zone of the lattice also has inversion symmetry. If the band is filled all pairs of orbitals  $\mathbf{k}$  and  $-\mathbf{k}$  are filled, and the total wavevector is zero. If an electron is missing from an orbital of wavevector  $\mathbf{k}_{e}$ , the total wavevector of the system is  $-\mathbf{k}_{e}$  and is attributed to the hole.

2. Energy:  $\epsilon_h(\mathbf{k}_h) = -\epsilon_e(\mathbf{k}_e)$ 

The energy of the hole is opposite in sign to the energy of the missing electron, because it takes more work to remove an electron from a low orbital than from a high orbital.



3. Velocity:  $\mathbf{v}_h = \mathbf{v}_e$ 

The velocity of the hole is equal to the velocity of the missing electron.

- Since  $\nabla \epsilon_h(\mathbf{k}_h) = \nabla \epsilon_e(\mathbf{k}_e)$ , so that  $\mathbf{v}_h(\mathbf{k}_h) = \mathbf{v}_e(\mathbf{k}_e)$ .
- 4. Effective mass:  $m_h = -m_e$

The effective mass is inversely proportional to the curvature  $d^2\epsilon/dk^2$ , and for the hole band this has the opposite sign to that for an electron in the valence band.



5. Equation of motion:  $\hbar \frac{d\mathbf{k}_h}{dt} = e(\mathbf{E} + \frac{1}{c}\mathbf{v}_h \times \mathbf{B})$ 

The equation of motion for a hole is that of a particle of positive charge *e*.



# **Electrons and Holes**

A full band can be written as: 
$$\sum_{i\neq +j}^{\pm N/2} \vec{k_i} + \vec{k_j} = 0 \text{ and } \sum_{i\neq +j}^{\pm N/2} \vec{k_i} = -\vec{k_j} = \vec{k_{-j}}$$

So when the state +j is empty in a band, the band has effective wavevector  $k_{-j}$ . Now the current flow in the incomplete band under the influence of a field E:  $\sum_{i=1}^{\pm N/2} (-e\vec{v}) - \sum_{i=1}^{\pm N/2} (-e\vec{v}) - e\vec{v} = 0$ 

$$\sum_{i=\pm 1}^{\pm N/2} \left( -e\vec{v}_i \right) = \sum_{i\neq +j}^{\pm N/2} \left( -e\vec{v}_i \right) - e\vec{v}_j = 0$$

$$e\vec{v}_i$$

This shows that an incomplete band (state +j empty) behaves just like a positive charge moving with the same velocity an electron would have in that state. Thus the properties of all of the remaining electrons in the incomplete band are equivalent to those of the vacant state j if the vacant state has:

- a. A k-vector  $k_{-j}$
- b. A velocity  $v_{+j}$  We call this vacant state a positive "hole" (h<sup>+</sup>)
- c. A positive charge +*e*

## **Dynamics of Electrons and Holes**

If this hole is accelerated in an applied electric field:  $m_h \frac{d\vec{v}_h}{dt} = +e\vec{E}$ 

The corresponding equation for the electron is:  $m_e \frac{d\vec{v}_e}{dt} = -e\vec{E}$ 

But earlier we deduced that the hole velocity is the same as that of the corresponding "missing" electron:  $\vec{v}_h = \vec{v}_e$ 

So by equating the derivatives we find: 
$$\frac{e\vec{E}}{m_h} = -\frac{e\vec{E}}{m_e} \longrightarrow m_h = -m_e$$

However, note that near the top of a band the band curvature is negative, so the effective electron mass is also negative. The corresponding hole mass is then positive.

So the equation of motion of a "hole" in an electromagnetic field is:

$$\vec{F} = \hbar \frac{dk_h}{dt} = e(\vec{E} + \vec{v}_h \times \vec{B})$$
 This explains why some metals have positive R<sub>H</sub>.

## **Carrier Mobility**

The mobility is the magnitude of the drift velocity of a charge carrier per unit electric field:  $\mu = |v|/E$ .

The electrical conductivity is the sum of the electron and hole contributions:

$$\boldsymbol{\sigma} = (ne\boldsymbol{\mu}_e + pe\boldsymbol{\mu}_h)$$

The drift velocity of a charge q was found to be  $v = q\tau E/m$ , whence

 $\mu_e = e au_e / m_e$  ;  $\mu_h = e au_h / m_h$  , where au is the collision time.

Crystal	Electrons	Holes	Crystal	Electrons	Holes
Diamond	1800	1200	GaAs	8000	300
Si	1350	480	GaSb	5000	1000
Ge	3600	1800	PbS	550	600
InSb	800	450	PbSe	1020	930
InAs	30000	450	PbTe	2500	1000
InP	4500	100	$\operatorname{AgCl}$	50	
AlAs	280		KBr (100 K)	100	—
AlSb	900	400	SiC	100	10-20

Table 3 Carrier mobilities at room temperature, in cm<sup>2</sup>/V-s

### p-n Junction

The most important building blocks in semiconductor devices are the p-n junction and the metal/semiconductor contact. In a p-n junction, we have a semiconductor crystal which is p-type on one side, and n-type on the other.



In the transition zone between the *n* and *p* regions, the Fermi level, i.e. the electrochemical potential, must therefore be common across the junction at thermal equilibrium, which therefore causes the band bending, as shown in the left panel **c**. The potential *V*(**x**) changes near the junction and, according to the Poisson equation, the macro-potential *V*(**x**) corresponds to a space charge  $\rho(\mathbf{x}) = \frac{\partial^2 V(x)}{\partial x^2} = -\frac{\varrho(x)}{\varepsilon \varepsilon_0}$ .

The diffusion voltage  $(V_D)$  is the difference between the maximum and minimum of V(x):

$$eV_{\rm D} = -(E_{\rm V}^n - E_{\rm V}^p) = \measuredangle T \ln \frac{p_p n_n}{n_i^2}$$

#### Assume the space-charge density is a step function:



Within the Schottky model, the lengths  $d_n$  and  $d_p$  give the spatial extent of the space-charge zone in the *n* and *p* regions, respectively. From charge neutrality it follows that

$$N_{\rm D}d_n=N_{\rm A}d_p\;,$$

and the continuity of V(x) at x = 0 demands

$$\frac{e}{2\varepsilon\varepsilon_0}(N_{\rm D}d_n^2 + N_{\rm A}d_p^2) = V_n(\infty) - V_p(-\infty) = V_{\rm D}$$

If the impurity concentrations are known, one can thus calculate the spatial extent of the space-charge layer

$$d_n = \left(\frac{2\varepsilon\varepsilon_0 V_{\rm D}}{e} \frac{N_{\rm A}/N_{\rm D}}{N_{\rm A}+N_{\rm D}}\right)^{1/2},$$

$$d_p = \left(\frac{2\varepsilon\varepsilon_0 V_{\rm D}}{e} \frac{N_{\rm D}/N_{\rm A}}{N_{\rm A} + N_{\rm D}}\right)^{1/2}$$

# **Modern Information Technology**

Modern information technology includes data processing (logic), data storage (memory) and data transfer in networks. More than 90% of the integrated circuits (ICs) are fabricated on silicon wafers and are used for data processing and storage. The reason is that the elemental semiconductor silicon permits the highest density of integration. As for data transfer, where optoelectronic devices such as detectors and optical emitters (light emitting diodes, LEDs, and lasers) are the fundamental elements. While the major material base for optoelectronic devices are the III-V semiconductors, and particularly GaAs. The reason is the much stronger coupling of electronic transitions to the electromagnetic field in the case of direct band gap III-V semiconductors is essential for optoelectronic devices.





## **Semiconductor diodes**

#### Diode-Di+ode

Di means two and ode means electrode. So physical contact of two electrodes is known as diode.

The important function for a diode is to convert an alternative current to a direct current.



#### Biased *p*-*n* Junction

If an external electrical voltage U is applied to a p-n junction, thermal equilibrium is destroyed, and the situation in the *p*-*n* junction can be described as a stationary state in the vicinity of thermal equilibrium. Because the space-charge zone between  $-d_p$  and  $d_n$ has a considerably higher electrical resistance than the region outside the *p*-*n* junction, the potential drop across the space-charge zone accounts for nearly all of the externally applied voltage U. Thus the potential drop across the space-charge zone,  $\rho(x)$ instead of being equal to the diffusion voltage  $V_{\rm D}$ , now has the value

$$V_n(\infty) - V_p(-\infty) = V_{\mathrm{D}} - U$$
.

One thus has

$$d_n(U) = d_n(U = 0)(1 - U/V_D)^{1/2}$$
,  
 $d_p(U) = d_p(U = 0)(1 - U/V_D)^{1/2}$ .



Let us consider the balance in the electron currents: we are concerned on the one hand with the drift currents of the minority carriers coming from the p region (where electrons are the minority carriers), which are drawn across into the n region. Because these minority carriers are continually generated in the p region by thermal excitation, this current is called the generation current,  $l^{gen}$ .

$$I^{\text{gen}} = I^{rec} \left( U = 0 \right)$$

This current is largely independent of the value of the diffusion voltage and therefore also of the external voltage.

The diffusion current of electrons from the *n* region, where the electrons are majority carriers, into the *p* region (called the *recombination current*  $I^{rec}$ ). With an externally applied voltage *U*:

$$I_n^{\rm rec} \propto e^{-e(V_{\rm D}-U)/\&T} = I_n^{\rm gen} e^{eU/\&T} ,$$

and therefore a total electron current  $I_n$  is

$$I_n = I_n^{\text{rec}} - I_n^{\text{gen}} = I_n^{\text{gen}} (e^{eU/\&T} - 1) .$$

#### The total current I(U) under applied external U is



## **Bipolar Transistors**

The classical bipolar transistor, invented by Bardeen, Brattain and Shockley in 1947, consists of two oppositely biased *pn*-junctions. Accordingly there are both, *npn*- and *pnp*-transistors; in *npn* devices the current is essentially carried by electrons, while in the *pnp* by holes.



A scheme of an *npn*-transistor together with its external circuitry is shown: the first  $n^+p$ -junction (emitter-base) is biased in the forward direction and the base-collector junction is biased in the reverse direction.



Emitter current  $I_{E}$ : the electrons emitted to the base  $I_{En}$  + the holes diffused from the base  $I_{Ep}$ .

Collector current  $I_{\rm C}$ : the electrons reached the base-collector junction  $I_{\rm Cn}$  + the thermally generated holes  $I_{\rm Cp}$ .

Base current  $I_{\rm B}$ : the holes diffused into the emitter  $I_{\rm Ep}$  + the portion  $I_{\rm Bn}$  due to the recombination of holes with electrons + the thermally generated holes  $I_{\rm Cp}$  from the collector region.

The collector current  $I_c$  is thus essentially controlled by the base current  $I_B$ .

Since the base current  $I_{\rm B}$  is typically below 1% of the collector current  $I_{\rm C}$  the collector current, which follows

$$I_{\rm C} \approx I_{\rm E} \approx A \frac{e D_n n_{\rm B}}{L_n} \left[ \exp(e U_{\rm EB} / \measuredangle T) - 1 \right].$$

Here, A is the current-carrying area of the transistor,  $D_n$  the diffusion constant of electrons in the base region,  $n_B$  their equilibrium concentration therein, and  $L_n$  the electronic diffusion length in the p-doped base.



Significant voltage and power amplification can be achieved since the input circuit of the transistor contains the low-resistance emitter-base pn junction biased in the forward direction, while the output current  $I_{\rm C}$  flows through the high resistance of the base-collector pn junction biased in reverse direction.

# **Field Effect Transistor (FET)**

A field effect transistor (FET) is essentially a resistor that is controlled by an external bias voltage. It consists of a current channel with two contacts, the source and the drain, and a third contact, the gate, which is separated from the current channel by an insulating barrier.



## Metal-Oxide-Semiconductor FET (MOSFET)



A bias voltage applied to the gate gives rise to a large electric field across the gate oxide layer that in turn causes a band bending in the underlying semiconductor. For a sufficiently positive gate voltage  $U_G$  the initial *p*-region (at  $U_G$ =0) below the gate is inverted ( $E_i < E_F$ ) to become an *n*-accumulation layer. The two source and drain *n*<sup>+</sup>-wells are thence connected by a conductive *n*-accumulation channel. The transistor has switched from ``off'' to ``on''.

## **Operation of nMOSFET**



Saturation mode at point of pinch-off

Saturation mode

## **Dynamic Random-Access Memory (DRAM)**

Dynamic random-access memory (dynamic RAM or DRAM) is a type of random-access semiconductor memory that stores each bit of data in a memory cell, usually consisting of a tiny capacitor and a transistor, both typically based on metal-oxide-semiconductor (MOS) technology. The capacitor can either be charged or discharged; these two states are taken to represent the two values of a bit, conventionally called 0 and 1. To prevent the leaking of electric charge on the capacitors, DRAM requires an external memory refresh circuit which periodically (every 64 ms) rewrites the data in the capacitors, restoring them to their original charge.

The main advantages of DRAM are its simple design, speed and low cost in comparison to alternative types of memory. The main disadvantages of DRAM are volatility and high power consumption relative to other options.

# **Operation of DRAM Reading**



- 1. The sense amplifiers are disconnected.
- 2. The bit-lines are pre-charged to exactly inbetween voltages.

BUS

- 3. The pre-charge circuit is switched off.
- 4. The desired row's word-line is driven to connect a cell's capacitor to its bit-line.

- 5. The sense amplifiers are now connected to the bit-lines pairs.
- 6. All storage cells in the open row are sensed simultaneously.
- 7. While reading is occurring, current is flowing back to recharge the cells.
- 8. When done with reading, the word-line is switched off to disconnect from the bit-lines.

# **Complementary MOS (CMOS)**

Nearly all advanced silicon ICs consist of combinations of *p*- and *n*-channel MOSFETs (CMOS = complementary MOS). Since one transistor of the MOSFET pair is always off, the combination of both transistor types allows the realization of logic gates that carry current only within the switching period between two bit-operations. CMOS circuits, therefore, consume less power, which is of significant advantage in the ever-increasing integration density.



Gate Voltage	n channel	p channel
High	On	Off
Low	Off	On

## **CMOS in Logic Gates**



The transistors' resistances are never exactly equal to zero or infinity, so the output will never exactly equal Vss or Vdd, but it will always be closer to Vss than A was to Vdd (or vice versa if A were close to Vss).

### **Semiconductor Heterostructures**

The structure consisting of layers of two different semiconductors grown epitaxially on one another is called a semiconductor heterostructure. Using modern epitaxial methods, such as molecular beam epitaxy (MBE) or metal organic chemical vapor deposition (MOCVD), it is possible to deposit two different semiconductors on one another in a crystalline form. It is also significant that using such epitaxial methods, ternary and quaternary alloys of the type Al<sub>x</sub>Ga<sub>1-x</sub>As or Ga<sub>x</sub>In<sub>1-x</sub>As<sub>y</sub>P<sub>1-y</sub> can be deposited, whose band gaps lie between those of the corresponding binary compounds.



It is possible to produce particularly good and defect-free epitaxial layers from semiconductors whose lattice constants match. It is expected that the two components GaAs and AlAs should lead to excellent crystalline quality with a variation of the band gap between 1.4 and 2.2 eV. In a heterostructure the band gap changes over distances of atomic dimensions. Two important points need to be considered for the electronic band structure of such a semiconductor heterostructure:

Semiconductor | Semicond



- a) How are the valence band edges  $E_v$  and  $E_{vac} = \begin{bmatrix} E_{vac} & I \\ X_{II} & I \end{bmatrix}$ conduction band edges  $E_c$  to be ``lined up''? This question addresses the so-called band discontinuity or band offset  $\Delta E_{v}$ .
  - b) What band bending occurs in the two semiconductors I and II to the left and right of the junction?

The most important material-related parameters of a semiconductor heterostructure are therefore the valence and conduction band discontinuities,  $\Delta E_{v}$  and  $\Delta E_{c}$ .

## **Band Offsets of Semiconductor Heterostructures**

Band offsets can well be explained by models in which the electronic bands in ideal, abrupt semiconductor heterostructures are lined up so that no atomic dipoles are created, due to electronic interface states or charge transfer in the chemical bonds at the interface. A detailed theoretical treatment of these models requires a microscopic description of the electronic properties of the few atomic layers at the semiconductor junction. A few experimentally determined valence band discontinuities  $\Delta E_v$  are listed below:

Hetero- structure	Valence band discontinuity $\Delta E_{\rm V}$ [eV]	Hetero- structure	Valence band discontinuity $\Delta E_{\rm V}$ [eV]	Hetero- structure	Valence band discontinuity $\Delta E_{\rm V}$ [eV]
Si-Ge	0.28	InAs-Ge	0.33	$CdTe-\alpha-Sn$	1.1
AlAs-Ge	0.86	InAs-Si	0.15	ZnSe-Ge	1.40
AlAs-GaAs	0.34	InP-Ge	0.64	ZnSe-Si	1.25
AlSb-GaSb	0.4	InP-Si	0.57	ZnSe-GaAs	1.03
GaAs-Ge	0.49	InSb-Ge	0.0	ZnTe-Ge	0.95
GaAs-Si	0.05	InSb-Si	0.0	ZnTe-Si	0.85
GaAs-InAs	0.17	CdS-Ge	1.75	GaSe-Ge	0.83
GaP-Ge	0.80	CdS-Si	1.55	GaSe-Si	0.74
GaP-Si	0.80	CdSe-Ge	1.30	CuBr-GaAs	0.85
GaSb-Ge	0.20	CdSe-Si	1.20	CuBr-Ge	0.7
GaSb-Si	0.05	CdTe-Ge	0.85		

# **Two-Dimensional Electron Gas (2DEG)**

For n-doping concentrations in AlGaAs of about  $10^{18}$  cm<sup>-3</sup>, typical thicknesses of the electron enrichment layer are in the region 50-100 Å in GaAs. The free electrons are confined in a narrow triangular potential well in the *z* direction, perpendicular to the heterojunction. The wave-function of such an electron thus has Bloch-wave character only parallel to the heterostructure; one speaks of a two-dimensional electron gas (2DEG). This so-called *z* quantization can be described straightforwardly using the time-independent Schrodinger equation with the potential *V*(*z*) and three effective mass components  $m_x^*$ ,  $m_y^*$ ,  $m_z^*$ . The total energy eigenvalues for such electron states quantized in the *z* direction

$$\mathbf{E} \begin{bmatrix} \mathbf{k}_{\parallel} &= \frac{\hbar^2 k_{\parallel}^2}{2 m_{\parallel}^*} + \varepsilon_j \text{ and } \varepsilon_j \simeq \frac{\hbar^2 \pi^2}{2 m_z^*} \frac{j^2}{d_z^2}, \quad j = 1, 2, 3 \dots \\ \text{These 2D subbands have a constant density of states } D(E) = \frac{dZ}{dE} = \frac{m_{\parallel}^*}{\pi \hbar^2} = \text{const} \\ \vdots & dZ = \frac{2\pi k \, dk}{(2\pi)^2} \text{ and } dE = \hbar^2 k \, dk/m_{\parallel}^* \end{bmatrix}$$

## **Semiconductor Superlattices**

A series of layers of semiconductors I and II with different band gaps can form a *composition superlattice*. If the distance between the potential wells is so small (less than 50-100 Å) that significant overlap between the wavefunctions exists, then this leads to a broadening of the bands.



The energetically lowest subband  $\varepsilon_1$  is noticeably broadened for  $d_z$  less than 50 A, and splits off as a band. For the higher subbands, the broadening begins at even larger  $d_z$  between the potential wells.

The broadening of the subbands and, in particular, the dependence of the subband energies on the spatial width of the potential wells is clearly seen in photoluminescence experiments. Photoluminescence spectroscopy is an important optical method for characterizing semiconductor heterostructures and superlattices. The semiconductor structure is illuminated with monochromatic laser light of photon energy above the band edge, thus creating electron-hole pairs. They occupy the subbands of the conduction or valence bands of the semiconductor or the corresponding excitonic states.



Local variations in the width of the quantum well lead to a spread in the spectral position of the emitted photoluminescence line. The energy difference between electron and hole subbands differs from the energy of the emitted photon by the amount of the exciton binding energy.

### **Semiconductor Laser**

The interaction of the electromagnetic light field with electronic excitations in the semiconductor involves three different processes: (i) an absorption of light quanta by an excitation of electrons from the valence band into the conduction band, or between electronic defect levels, (ii) an inverse stimulated emission, in which the incident photon stimulates an electron to be de-excited from a state of higher energy, and (iii) the spontaneous emission of light. All three types of interactions are used in photoelectric devices: For photodetectors and solar cells the absorption process is essential. Light emitting diodes (LED) utilize spontaneous emission, while lasers are based on stimulated coherent emission. In all three processes energy conservation is obeyed  $\hbar\omega = E_2 - E_1$ 



Under stationary illumination with light the transition rates must compensate each other. With  $A_{21}$  as the probability for spontaneous emission and  $B_{21}$  and  $B_{12}$  the probabilities for stimulated emission and absorption, respectively, then

$$\dot{n}_2 = -B_{21}n_2|\mathscr{E}(\omega)|^2 - A_{21}n_2 + B_{12}n_1|\mathscr{E}(\omega)|^2 = -\dot{n}_1 = 0$$

Neglecting spontaneous emission and using the equality  $B_{21} = B_{12}$  one obtains the following laser condition:

$$\dot{n}_2 = -(n_2 - n_1)B_{21}|\mathscr{E}(\omega)|^2 < 0$$
 i.e.  $n_2 > n_1$ .

This condition describes the inversion of the occupation statistics.

In a semiconductor laser, population inversion is achieved by ``pumping'' into the excited state. A convenient method for pumping is to bias a *p*-*n* junction in the forward direction and to flood the space-charge region with nonequilibrium electrons and holes. The laser condition has to be modified for the valence band ( $E_v$ ) and the conduction band states ( $E_c$ ) of a semiconductor laser. Here, the transition rates are proportional to the density of occupied initial and the density of empty final states. The condition for population inversion in a *p*-*n* junction biased in the forward direction thus follows as

$$-B_{21}D_{\rm C}(E_{\rm C})D_{\rm V}(E_{\rm V})f(E_{\rm C})[1-f(E_{\rm V})] +B_{12}D_{\rm V}(E_{\rm V})D_{\rm C}(E_{\rm C})f(E_{\rm V})[1-f(E_{\rm C})] < 0.$$

The occupation probabilities can be approximated by Fermi functions with  $E_{F}^{n}$ ,  $E_{F}^{p}$  as quasi-Fermi levels in the *n*- and *p*-region, respectively

$$f(E_{\rm C}) = [1 + \exp(E_{\rm C} - E_{\rm F}^n)/kT]^{-1}$$
,  $f(E_{\rm V}) = [1 + \exp(E_{\rm V} - E_{\rm F}^p)/kT]^{-1}$ 

Hence, the condition for population inversion becomes

$$E_{\rm F}^{n} - E_{\rm F}^{p} > E_{\rm C} - E_{\rm V} = E_{\rm g}$$
.

For laser operation, the quasi-Fermi levels in the *n*- and *p*-doped region must energetically be separated from each other by more than the band gap energy  $E_g$ . The *p*- and *n*-type regions, therefore, have to be doped deep into degeneracy.



For laser operation, the quasi-Fermi levels in the n- and p-doped region must be separated from each other by more than the band gap energy  $E_{g}$ .

#### **Double Heterostructure Laser**



A lowly doped or intrinsic i-GaAs layer is inserted between
two low *p*- and *n*-doped wide band gap AlGaAs regions.

In spite of the low doping in the AlGaAs regions, the quasi-Fermi levels are located within the conduction and valence band of the active i-GaAs layer under strong forward bias. Population inversion is thus easily achieved. The active i-GaAs region is flooded with non-equilibrium electrons and holes, which are confined to the active region by the conduction and valence band discontinuities, respectively. This effect, which causes enhanced light emission is called ``electrical confinement''. Additionally, this structure provides an ``optical confinement'', since the refractive index of the active GaAs region exceeds that of the adjacent AlGaAs layers. The light originating from stimulated emission in the GaAs is therefore totally reflected at the AlGaAs layers.

An important figure of merit of a semiconductor laser is a threshold current as low as possible arising from low losses in the resonator. Population inversion is easily achieved in the double heterostructure laser comparing to the highly doped, degenerate semiconductors of the high current densities.



further figure of merit Α of а semiconductor laser is the critical temperature  $T_0$ , which should be as high possible because the threshold as current  $I_{th}$  depends exponentially on the working temperature T as  $I_{\text{th}} \propto \exp(T/T_0)$ , so that the laser shows a slow increase of the threshold current with working temperature T. Typical values of  $T_0$  are close to 100°C for double heterostructure lasers.

Further improvement of the laser performance is achieved by incorporating into the active zone even thinner layers of a semiconductor with a lower band gap than that of GaAs, which can render the *quantum-well lasers*.

## Problems

- 1. A semiconductor with a band gap energy  $E_g$  of 1eV and equal hole and electron effective masses  $m_e^* = m_h^* = m_0$  ( $m_0$  is free electron mass) is p-doped with an acceptor concentration of  $p = 10^{18}$  cm<sup>-3</sup>. The acceptor energy level is located 0.2eV above the valence band edge of the material.
  - a) Show that intrinsic conduction in this material is negligible at 300 K.
  - b) Calculate the conductivity  $\sigma$  of the material at room temperature (300 K), given a hole mobility of  $\mu_p = 100 \text{ cm}^2/\text{Vs}$  at 300 K.
  - c) Plot the logarithm of the hole concentration,  $\ln p$ , versus reciprocal temperature 1/T for the temperature range 100 to 1000 K.
- 2. A silicon *p*-*n* step junction diode is doped with  $N_d = 10^{16} \text{ cm}^{-3}$  and  $N_a = 4 \times 10^{18} \text{ cm}^{-3}$  on the *n* side and *p* side, respectively. Calculate the build-in potential, space charge width, and maximum field at zero bias at room temperature.

- **3.** (a) Sketch the energy band diagram for a *p*-*n*-*p* transistor at equilibrium and under the normal active mode of operation.
  - (b) Sketch a schematic diagram to represent the transistor and indicate all current components.
  - (c) Write down the relative equations for the currents of emitter, base and collector.
- **4.** Try to realize the NOR logic gate in CMOS circuits and use the truth table to confirm the operations.